SELF-SIMILAR PLANE MOTIONS OF A HEAT-CONDUCTING GAS HEATED BY RADIATION

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Processes occurring in a substance subjected to radiation from optical quantum generators have recently attracted considerable attention from researchers. Of considerable importance are thermodynamic processes. In fact, since the absorption coefficient depends on temperature and density, motion, accompanied by temperature and density variation, materially affects transmissivity and rate of heating. The pattern of motion is very complex, even in one-dimensional (plane) problems, particularly in vapors having generated shock waves propagating from the region of energy release. Their range and effects in these processes vary with the intensity of the incident radiation flux and the initial density of the substance.

At high vapor temperatures, considerably exceeding the sublimation temperature T_s of the substance (at which the internal energy per unit of mass is greater than the heat of evaporation Q_s and the vapor density ρ is appreciably lower than the density ρ_0 of the solid body), the problem can be simplified by assuming

$$T_s = Q_s = 0, \quad \rho_0 = \infty \quad (v_0 = 1 / \rho_0 = 0). \tag{0.1}$$

In the region of multiple and full ionization, the absorption coefficient k_q of optical radiation of ionized vapors can usually be described by a power function of the pressure p and the specific volume v,

$$k_q = K_q v^a p^b \tag{0.2}$$

(for a fully ionized gas $\alpha = -5/2$ and $b = -3/2$).

The heating process of a perfect nonheat-conducting gas and gas motion were considered in [1-3] on assumptions (0.1) and (0.2). The heating of a gas adjoining a vacuum results in an increase of pressure p, and its consequent scatter. Decrease of the absorption coefficient k_q with decreasing density and increasing temperature T produces a deeper penetration of radiation into matter. The heating and motion of an absolutely cold infinitely dense gas is a self-similar problem. It was considered in approximation in [3]. Here a detailed analysis is made of the ordinary differential-equation system defining the self-similar motion (2-5), and results of numerical integration are given (8) for: the distribution of parameters of the maximum temperature T_m attained during heating; the pressure p_v at the body surface resulting from the scatter of vapors; the velocity u_0 of the gas boundary. The two latter parameters can be most conveniently used for the indirect determination of attained temperature.

As shown by Nemchinov, heating and rarefaction waves may also occur when the forward front of the vapor-heating wave does not coincide with the evaporation front, i.e., as if the ionized vapor layer had been generated prior to the considered interaction phase. This often takes place under laboratory conditions in experimental determination of the effect of the OQG (optical quantum generator) on a solid-body surface, e.g., when a "gigantic impulse" is preceded by a prolonged "phone" of continuous radiation of a less powerful flux.

Moscow. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, Vol. 9, No. 4, pp. 18-28, July-August, 1968. Original article submitted September 27, 1967.

In such problems it is necessary to consider the motion of gas forward of the heating wave front. We assume, for simplicity, that the temperature and the rate of motion of vapors are small in comparison with those of the initial impulse and that the initial vapor pressure is the same throughout. A heating wave, whose dimension (of the order of $1/p\,k_q$) increases owing to the decrease of k_q with increasing temperature, propagates during the first phase of radiation heating of the gas which is virtually stationary and its density has not yet been altered [1, 2]. Heating increases gas pressure and, when the gas is contiguous to a vacuum, results in gas dispersion. Initially, the rarefaction wave occupies a small part of the heated region, and the light flux q in it does not appreciably vary in comparison with the incident flux q_0.

The presence of a rarefaction wave has virtually no effect on the nearly stationary gas forward of its front. This phase was considered in [2]. After the rarefaction wave has moved through a distance comparable to the dimensions of the whole heated region, the decrease of the absorption coefficient k_q, consequent on the decrease of density ρ, which leads to increased radiation penetration, becomes important, and a self-consistent rarefaction and heating wave sets in [3].

The reaction force accompanying gas dispersion generates a shock wave which propagates in the substance. Since in this problem the shock-wave motion vitiates, generally speaking, its self-similarity, the results relative to the shock-wave formation stage cited in [3] had to be obtained by the approximate-difference method. The shock wave becomes subsequently detached from the heated zone, so that the self-similar solution [3], obtained without taking into consideration the shock wave, can be used.

As shown in this paper, the problem will be self-similar for finite initial density ($\rho_0 < \infty$) of the heated substance and for the shock wave taken into account, if the light flux variation is a power function of time, such that the characteristic gas density is constant ($q \sim t^{3/2}$ for $a=-5/2$ and $b=-3/2$). This problem is considered in detail in § 6 and 7.

The self-similar solution considered defines the processes of heating and gas motion from the very beginning, hence, the consecutive changes of phase, in which individual processes are unessential, do not appear in it.

The characteristic density of dispersed matter varies with the radiation flux (or the initial density). This makes it possible to trace the effect of parameter variation on the solution transition for density close to initial to the limit solution for a density considerably lower than the initial, on the distribution of parameters, and on shock wave amplitude and position. The derived solution is, in itself of interest in the determination of parameters for plasma heated by an optical quantum generator when, during the initial phase of impulse, the radiation flux increases.

In the problems considered above, thermal conductivity was not taken into account.

It follows from [4, 5] that at sufficiently high temperatures the effect of electron thermal conductivity becomes appreciable. The problem of heat-conducting gas motion with a nonlinear thermal-conductivity coefficient was considered in [6-8] but without taking into account the heat added by radiation.

In this paper self-similar motions are considered with these two factors also taken into account.

The self-similarity condition implies that for $k_q \sim p^{3/2}$ the thermal conductivity $k_f \sim p^2$ (p is the pressure). This dependence $k_f(p)$ is close to the true one in fully ionized plasma ($k_f \sim p^{5/2}$). Hence the solution derived in § 7 can be used for approximately assessing the thermal-conductivity effect.

Because of the formal generality of problems considered here, the analysis of the set of problems (§ 2-5) with and without taking into account thermal conductivity is carried out concurrently.

1. For a perfect gas with constant adiabatic exponent the equations of motion, continuity, energy, and of transport of light and heat fluxes are of the form

$$\frac{\partial u}{\partial t} + \frac{\partial p}{\partial m} = 0, \quad \frac{\partial v}{\partial t} = \frac{\partial u}{\partial m}, \quad v\frac{\partial p}{\partial t} + \gamma p \frac{\partial v}{\partial t} = (1-\gamma)\frac{\partial (f+q)}{\partial m}$$

$$f = -k_f \frac{\partial (pv)}{\partial m} = -K_f p^c v^d \frac{\partial (pv)}{\partial m}, \quad \frac{\partial q}{\partial m} = -k_q q = -K_q v^a p^b q. \tag{1.1}$$

Here u is the velocity; v is the specific volume; p is the pressure; q and f are, respectively, the light and heat fluxes; t is the time; m is the Lagrangian mass coordinate; k_f and k_q are, respectively, the thermal conductivity and the absorption coefficient; K_f and K_q are numerical coefficients.

For a fully ionized plasma the exponents are: $a=-5/2$, $b=-3/2$, $c=5/2$, and $d=3/2$.

At the initial instant of time $t=0$ the gas in the half-space $m \geq 0$ is assumed to be cold and stationary

$$u = p = 0, \quad v = v_0 \qquad \text{for } t = 0. \tag{1.2}$$

At $t \geq 0$ radiation q impinges on the gas, a heat flux f penetrates its boundary, and the gas is set in motion by a piston whose velocity u (or the pressure p exercised by it on the gas) is specified. The related boundary conditions of the problem are defined by three functions

$$q(0, t), \qquad f(0, t), \qquad u(0, t) \qquad (\text{or} \quad p(0, (t)). \tag{1.3}$$

Equations (1.1)-(1.3) will be self-similar, if conditions

$$\begin{gathered} q(0, t) = q_0 t^{-3/k}, \quad u(0, t) = u_0 t^{-1/k} \quad (\text{or} \quad p(0, t) = p_0 t^{-2/k}) \\ f(0, t) = f_0 t^{-3/k}, \quad v(m, 0) = v_0, \quad k = 1 + 2b, \quad c = {}^1/_2 - b, \end{gathered} \tag{1.4}$$

are satisfied.

When $v_0=0$ (the limit case of heating a dense medium), the problem becomes self-similar, under conditions less rigid than (1.4), with respect to the light and heat fluxes

$$f(0, t) = f_0 t^g, \qquad q(0, t) = q_0 t^g \qquad (g \text{ — is an arbitrary number}). \tag{1.5}$$

Thus, at the piston, the heat flux must be proportional to the incident radiation flux. For a thermally insulated piston ($f_0=0$) this constraint is removed.

In what follows (§1-7), the motions will be considered on the assumption that (1.4) is satisfied. We introduce self-similar variables V, P, U, Q, F, and x defined by

$$\begin{gathered} v(m, t) = v_0 V(x), \quad p(m, t) = t^{-2/k} v_0^{(1-2a)/k} K_q^{-2/k} P(x), \\ u(m, t) = t^{-1/k} v_0^{(1+b-a)/k} K_q^{-1/k} U(x), \\ q(m, t) = t^{-3/k} v_0^{(2-3a+e)/k} K_q^{-3/k} Q(x), \\ f(m, t) = t^{-3/k} v_0^{d+1+({}^3/_2-2a+2ab)/k} K_f K_q^{-(1+2c)/k} F(x), \\ x = m t^{-2b/k} K_q^{1/k} v_0^{(a+b)/k}. \end{gathered} \tag{1.6}$$

and by substituting these into (1.1), obtain a system of self-similar equations

$$\begin{gathered} V' = \frac{1}{\delta}\left(\frac{F}{P^c V^d} - \frac{UV}{k}\right), \quad P' = -r^2 x^2 V' + \frac{U}{k}, \\ \sigma F' = (\gamma - 1) Q V^a P^b + 2PV/k + rx(VP' + \gamma P V'), \\ U' = -rxV', \quad Q' = -QV^a P^b, \quad \delta = P - r^2 x^2 V, \\ r = 2b/k, \quad \sigma = (\gamma - 1) K_f K_q v^{a+d+1/2}. \end{gathered} \tag{1.7}$$

In the absence of heat conduction ($k_f=0$) the system of equations is of the form

$$\begin{gathered} V' = \{[(1-\gamma) QV^a P^b - 2PV/k] / rx - UV/k\} / \delta, \\ U' = -rxV', \qquad P' = -r^2 x^2 V' + U/k, \\ Q' = -QV^a P^b, \qquad \delta = \gamma P - r^2 x^2 V. \end{gathered} \tag{1.8}$$

2. It follows from the initial and the boundary conditions (1.2) and (1.3) that the number (6) of boundary conditions exceeds the number (5) of equations. For (1.7) these boundary conditions can be written in the form

$$Q = Q_0, \quad P = P_0 \quad (\text{for } U = U_0), \quad F = F_0 \quad \text{for } x = 0 \quad U = P = 0, \quad V = 1 \quad \text{for } x = \infty. \tag{2.1}$$

For x=0 the condition $F = F_0$ does not apply to Eqs. (1.8). To solve the stated boundary-value problem and satisfy the "redundant" boundary condition, it is necessary to introduce a certain free parameter. Generally the latter can be either the self-similar coordinate x_2 or, with a continuous solution, the jump $[\varphi^{\dagger}]$ of one of the derivatives.

Integration of Eqs. (1.7) can be carried out from x=0 to $x = x_1$ by selecting the three free parameters U(0) (or P(0)), V(0), and x_2 to satisfy the last three of conditions (1.9). Such a method would, however, be too laborious, since it necessitates the determination of three unknown parameters for obtaining a single solution. Integration from $x = x_1$ to x=0 is more effective. In this case any of the intermediate solutions will be a solution of the input problem with boundary parameters at point x=0, which, however, may differ from the required P_0, Q_0, and F_0. The search for a solution for a priori specified P_0, Q_0, and F_0 will provide solutions for the whole range of these parameters, and the computation of equations would provide additional useful information on the kind of motions considered here.

3. We shall prove that continuous solutions of this problem are not possible. By virtue of the boundary conditions (2.1) the sign of function $\delta(x) = P - r^2x^2V$ changes in the interval $(0, \infty)$. We denote by x_p the point at which $\delta(x) = 0$, and add subscripts p to all parameters at that point. Let us assume that this point is not a singular one, i.e.,

$$kF_p + U_pV_p^{d+1}P_p^{\,c} \neq 0 \qquad \text{for } rx_p = (P_p / V_p)^{1/2}. \tag{3.1}$$

In its neighborhood functions V, P, U, Q, and F are (to within terms of a higher order of smallness) of the form

$$V = V_p + v, \quad P = P_p - r^2x^2v, \quad U = U_p - rx_pv$$
$$Q = Q_p - Q_pV_p^{\,a}P_p^{\,b}\,(x - x_p), \quad F = F_p + rx_p\,(\gamma P_p - r^2x_p^2V_p)v. \tag{3.2}$$

Substituting (3.2) into the first of Eqs. (1.7) and, again, neglecting terms of a higher order of smallness, we obtain

$$v\,\frac{dv}{dx} = \frac{kF_p + U_pV_p^{d+1}P_p^{\,c}}{2r^2x_p^2P_p^{\,c}V_p^{\,d}} = A_1. \tag{3.3}$$

The following relationships

$$\left.\frac{dx}{dv}\right|_{x=x_p} = A_1v\,|_{x=x_p} = 0, \quad \left.\frac{d^2v}{dx^2}\right|_{x=x_p} = A_1 \neq 0, \tag{3.4}$$

mean that the variable x and the function v have extrema at point x_p, i.e., function v(x) exists only on one side of $x = x_p$. It follows from this that when there are no nodal-type singular points in the interval $(0, x_1)$ the solution will be discontinuous.

A similar reasoning, with only slight changes, proves the absence of continuous solutions which do not pass through a singular point within the interval $(0, x_1)$, also in the case of Eqs. (1.8) $(k_f = 0)$. We then have $rx_p = (\gamma P_p / V_p)^{1/2}$.

4. The relationships at discontinuities at point $x = x_2$ can be obtained directly from the self-similar equations (1.7) by integrating these from $x_2 - \Delta x$ to $x_2 + \Delta x$ and passing to limit for $\Delta x \to 0$. After simple transformations, we obtain formulas similar to those in [6]

$$V_2 = P_1 / r^2 x_2^2, \quad P_2 = P_1 V_1 / V_2, \qquad Q_2 = Q_1,$$
$$U_2 = U_1 + r x_2 (V_1 - V_2),$$
$$F_2 = F_1 + (\gamma - 1) r^3 x_2^3 (V^2_2 - V^2_1) / 2\sigma. \tag{4.1}$$

The same operation applied to Eqs. (1.8) yields

$$V_2 = V_1 (\gamma - 1) / (\gamma + 1) + 2\gamma P_1 / (\gamma + 1) r^2 x_2^2, \ Q_2 = Q_1,$$
$$P_2 = P_1 + r^2 x_2^2 (V_1 - V_2), \qquad U_2 = U_1 + r x_2 (V_1 - V_2). \tag{4.2}$$

Subscripts 1 and 2 in (4.1) and (4.2) denote parameters to the left and right of point x_2, respectively. We note that in both cases δ undergoes a change of its sign when passing through the discontinuity

$$\delta_1 = -\delta_2. \tag{4.3}$$

This ensures a stepwise transition of δ (x) from the region $\delta > 0$ at $x < x_2$ to region $\delta < 0$ at $x > x_2$, as dictated by the boundary conditions (1.9), thus avoiding point x_p at which $\delta = 0$.

Let us prove the impossibility of existence of more than one discontinuity in the solution, if it does not pass through point x_p. To do this we rewrite (4.1) and (4.2) for specific volumes in the form

$$V_2 = V_1 + \delta_1 / r^2 x_2^2 \quad \text{for } k_f \neq 0,$$
$$V_2 = V_1 + 2\delta_1 / (\gamma + 1) r^2 x_2^2 \quad \text{for } k_f = 0, \tag{4.4}$$

expressing δ_1 in terms of P_1 and V_1

$$\delta_1 = P_1 - r^2 x_2^2 V_1 \quad \text{for } k_f \neq 0,$$
$$\delta_1 = \gamma P_1 - r^2 x_2^2 V_1 \quad \text{for } k_f = 0. \tag{4.5}$$

Let us assume that two discontinuities exist at points $x = x_2$ and $x = x_3$ when $x_2 < x_3$. According to (2.1) $\delta_1(x_2) > 0$, hence, at point x_2 we have, in accordance with (4.4), a compression jump $V_2 > V_1$. Point x_3 will be reached for $\delta_1(x_3) < 0$ (in accordance with (4.3)). We will, consequently, have at point x_3 a rarefaction jump $V_2 < V_1$, which violates the second law of thermodynamics.

A definite relation exists between the parameters in a solution passing through the singular point x_p (vanishing of the numerator in the expressions for V' in (1.7) and (1.8)), and this cancels one free parameter. When the singular point x_p is not a saddle, there must necessarily exist one more discontinuity. However, the analysis of variation of the numerator in the expression for V' in (1.8) at point x_p with varying free parameter x_1, carried out by a qualitative examination of integral-curve behavior of (1.8) and of obtained numerical results, has shown that in the region bounded by negative values the variation of this parameter is monotonic. The assumption of existence of solutions passing through the singular point x_p and satisfying the boundary conditions (1.9) is thus shown to be unfounded, although we have no strict proof of this.

5. Let us consider now the singular points of Eqs. (1.7) and (1.8). As shown in §4, function δ(x) does not vanish in the interval (0, x_1); hence, singularities can only appear at points $x = 0$ or $x = x_1$ of the interval boundaries.

For $P_0 > 0$ Eq. (1.7) does not have a singularity at point $x = 0$, while Eqs. (1.8) allow at this point a relationship clearly showing that point $x = 0$ is singular. To find the latter, we write the nonself-similar energy equation from (1.1) for m=0 (with $k_f = 0$)

$$v(0, t) \frac{dp(0, t)}{dt} + \gamma p(0, t) \frac{dv(0, t)}{dt} = (\gamma - 1) K_q v^a (0, t) p^b (0, t) q(0, t), \tag{5.1}$$

substitute in it the expressions given in (1.6) for p, v, and q, and take into consideration that at point m=0 we have x=0.

After trivial reductions Eq. (5.1) becomes readily integrable. As a result we obtain

$$k(\gamma - 1) Q_0 = -2V_0^{1-a} P_0^{1-b}. \tag{5.2}$$

Reverting to Eqs. (1.8), we ascertain that point x=0 is singular. The solution in the neighborhood of this point is of the "node" kind, and, within terms of higher order of smallness, is of the form

$$V = V_0 + \frac{2P_0^{1+b}V_0^{1+a} + U_0V_0(2bk^{-1} - 2k^{-1} - r)}{2P_0(a-1) - 2b\gamma P_0} x + |A|\, x\, |^{(a-1)/b\gamma},$$
$$U = U_0 - rx(V - V_0), \qquad P = P_0 + U_0 x / k, \qquad Q = Q_0(1 - V_0^a P_0^b x), \tag{5.3}$$

where A is an arbitrary constant.

In dispersion into vacuum (P_0=0) point x=0 for Eqs. (1.7) and (1.8) is always a singular point of the node kind. The solution of Eqs. (1.8) in the neighborhood of this singular point is, within terms of higher order of smallness, of the form

$$V = \left[Ax^\theta - \frac{(\gamma - 1) Q_0 (U_0/k)^{b-1} x^{b-1}}{r[1 + \gamma(b-1)/(1-a)] + 2/k}\right]^{1/(1-a)},$$
$$U = U_0 + rxV(1-b)/(b-a), \qquad P = U_0 x / k,$$
$$Q = Q_0\left[1 - \left(\frac{U_0}{k}\right)^b x^{b+1} V^a \left(\frac{1-a}{b-2a+1}\right)\right] \qquad \theta = \left(\frac{2}{k} + r\right)\frac{a-1}{\gamma r}, \tag{5.4}$$

where A is an arbitrary constant.

The nature of the singular point x=0 (at P_0=0) is the same for (1.7) and (1.8) when the exponent d< 0. The case of Q_0=0 and d> 0 was considered in [7]. Analysis of the singular point x=0 for arbitrary a, b, c, and d is made difficult by the great number of possible variations.

System (1.8) has a singularity at the terminal point x=x_1 at which U=P=Q=0 and V=V_1. The only solution issuing from this point is in its neighborhood, within terms of higher order of smallness, of the form

$$P = [-bV_1^a \ (x_1 - x)]^{-1/b}, \qquad V = V_1 - P / r^2 x_1^2,$$
$$U = P / rx_1, \qquad Q = xr_1V_1 P / (\gamma - 1), \quad (x < x_1, b < 0). \tag{5.5}$$

In the presence of heat conduction ($k_f \neq 0$) it is also possible to speak of a singularity at a certain point x=x_1 at which U=P=Q=F=0 and V=V_1. In the neighborhood of this point the solution, which is also of the form (5.5), is to be supplemented by the expression for the heat flux which, for $\alpha = -5/2$ and b=−3/2, is of the form

$$F = {}^3/_2 (x_1 - x). \tag{5.6}$$

However, since (unlike the case of k_f=0) we have at our disposal only two free parameters x_1 and x_2 (when integrating equations from x=x_1 to x=0), the input problem (§1) cannot be solved for any set of P_0, Q_0, and F_0 specified for x=0.

When solving the input problem numerically it is expedient to make use of the fact that, depending on the relation between P_0, Q_0, and F_0, there exists a terminal point x_1 such that for $x \geq x_1$ $Q(x) \ll F(x)$ (or $F(x) \ll Q(x)$). We can then assume in approximation that in the neighborhood of this point $Q(x) \equiv 0$ (or $F(x) \equiv 0$), and proceed from the singular point x_1, using the appropriate power formulas and taking as the third free parameter (apart from the coordinates of the singular point x_1 and of x_2 of the strong discontinuity) point x_3 at which $Q(x_3) = \varepsilon$ (or, respectively, $F(x_3) = \varepsilon$), where $\varepsilon \ll F(x_3)$ (or $\varepsilon \ll Q(x_3)$).

6. Light or heat fluxes of sufficiently high intensity reaching a gas generate patterns of motion in which the variation of the specific volume V(x) relative to V(x_1) in the interval (x_2, x_1) and of the interval

$(0, x_2)$ relative to $(0, x_1)$ can be neglected; i.e., it is possible to consider the heated gas as stationary throughout the interval $(0, x_1)$. The exact analytical solution of this problem was derived in [1, 2] for absorption of a light flux with nonlinear absorption coefficient. Similar patterns exist, also, when the light flux is absorbed by a heat-conducting gas. For $V \equiv 1$ the self-similar equations (1.7) are of the form

$$rxP' = (\gamma - 1) Q' - \frac{2P}{k} - K_f \frac{d}{dx}\left(P^c \frac{dP}{dx}\right), \qquad Q' = - P^b Q. \tag{6.1}$$

When $c=-b$, (6.1) admits particular solutions of the form

$$P = P_0 \left(1 - \frac{x}{x_1}\right)^{-1/b}, \qquad Q = Q_0 \left(1 - \frac{x}{x_1}\right)^{-1/b}, \tag{6.2}$$

$$x_1 = -\frac{P_0^{-b}}{b}, \qquad Q_0 = \left[\frac{2P_0}{k} - \frac{K_f P_0^{1-b}}{(bx_1)^2}\right] \frac{bx_1}{\gamma - 1}. \tag{6.3}$$

In spite of their very particular form, solutions (6.2) give a good general picture of the situation arising in the presence of heat conduction – the increased light flux Q_0 necessary for obtaining the maximum of temperature P_0 (here $V=1$).

In the case considered this dependence can be written in the form

$$Q_0(K_f) = A + BK_f \qquad \left(A = -\frac{2P_0^{1-b}}{k(\gamma - 1)}, \quad B = \frac{P_0}{\gamma - 1}\right). \tag{6.4}$$

In the absence of heat conduction ($k_f=0$), functions (6.2) are exact solutions (see, e.g., [1, 2]) of the self-similar input problem, and provide a simple expression for the flux Q_0 necessary for obtaining maximum temperature P_0

$$Q_0(P_0) = \frac{2P_0^{1-b}}{(1-\gamma)(1+2b)}. \tag{6.5}$$

For a fully ionized plasma ($b=-3/2$ and $\gamma=5/2$) from (6.5) we have

$$Q_0 = {}^3/_2 P_0^{5/2}. \tag{6.6}$$

Numerical calculations carried out for the region $Q_0 > 10$ ($V(x_1)=1$) have yielded the same result, thus confirming the existence of a wide range of values of Q_0 in which gas heating is defined with a considerable degree of accuracy by simple relationships (6.2) when $k_f=0$.

The other limit mode, opposite to that described above, occurs in the input problem when

$$V_1 \to 0 \quad \text{for } Q_0 = \text{const}, \ P_0 = \text{const}, \ F_0 = \text{const}. \tag{6.7}$$

Here the transformation formulas (6.6) must be used in the form

$$\begin{gathered} q = q_0 t^{-3/k} Q, \qquad v = q_0^{k/n} K_q^{3/n} V, \\ p = q_0^{(1-2a)/n} K_q^{-1/n} t^{-2/k} P, \qquad u = q_0^{(1+b-a)/n} K_q^{1/n} t^{-1/k} U, \\ f = K_f K_q^{\frac{3}{n}\left(d+1+\frac{1,5-2a+2ab}{k}\right) - \frac{2c+1}{k}} q_0^{\frac{(d+1)k+1,5-2a+2ab}{n}} t^{-\frac{3}{k}} F, \\ x = m t^{-2b/k} q_0^{(a+b)/n} K_q^{2/n}, \qquad n = 2 - 3a + b. \end{gathered} \tag{6.8}$$

A solution of the stated boundary value problem exists for $V_1=0$ and is an analog of the self-similar solution for time-dependent flux $q(0, t) \sim t^{-3/k}$ [3].

Self-similar profiles of this solution are shown in Fig. 1 for $P_0=0.18$, $Q_0=6.65$, and $k_f=0$. The extension of the high pressure region to $x=\infty$ is explained by the fact that in the mass the speed of sound

$$c_m = (\gamma P / V)^{1/2} \to \infty \text{ for } V \to 0.$$

The respective self-similar profiles for $V_1 \ll 1$ obviously do not differ greatly from those for $V_1=0$; however, owing to the finite speed of sound c_m when $V>0$, the pressure $P>0$ does not "penetrate" point $x=\infty$, but vanishes at the terminal point x_1.

Typical for the transition (6.7) to limit are the following relationships:

$$x_1 - x_2 \to 0, \quad x_1 \to \infty \quad \text{for} \quad V_1 \to 0. \tag{6.9}$$

In the interval (x_v, x_1), the solution of such self-similarproblems can be approximately given in the form

$$P(x) = P_v, \quad V(x) = V_1, \quad U = Q = 0 \quad \text{for } x_v \leqslant x \leqslant x_1, \tag{6.10}$$

omitting at point x_v the solution of Eqs. (1.8) with initial conditions

$$P = P_v, \quad V = V_1, \quad U = Q = \varepsilon \ll 1 \quad \text{for} \quad x = x_v. \tag{6.11}$$

Here P_v and x_v are the two free parameters (instead of x_1 and x_2), with which the two left-hand boundary conditions in (6.7) can be satisfied.

Because of the closeness of the solution for $V_1 \ll 1$ to the limit mode at $V_1=0$, it is possible to define the relation between the shock-wave coordinate x_2 and V_1

$$V_1 = (\gamma + 1) \; P_m / 2r^2 x^2. \tag{6.12}$$

Here P_m is the maximum pressure in the limit problem at $V_1=0$.

7. Analysis of the above modes and the results of calculations on a computer for intermediate modes leads to the conclusion that the solutions of the boundary-value problem stated in § 1 can be found in the class of functions satisfying the relationships at the shock wave, with a single discontinuity.

Let us consider the set of solutions of (1.8)-(1.9) (omitting heat conduction) with boundary conditions $Q_0=\text{const}$, $P_0=\text{const}$, and $0 \leq V_1 \leq \infty$. Modes close to the limits ($V_1 \ll 1$ and $V_1 \gg 1$) were described in §6. The self-similar profiles of intermediate modes ($0 < V_1 < \infty$) are of the same pattern, consisting of two continuous curves along segments $[0, x_2]$ and $[x_2, x_1]$ with a transition jump from one to another at point x_2, as defined by (4.2). With increasing V_1 point x_2 is shifted towards $x=0$, the shock wave amplitude diminishes, and the length of segment $[x_2, x_1]$ increases. With decreasing V_1 the coordinate x_2 of the strong discontinuity and the shock-wave amplitude increase, while the length of segment $[x_2, x_1]$ vanishes.

The similarity between the heat waves TVI and TVII in [6], and the modes close to the limits $V_1=\infty$ and $V_1=0$ can be readily seen.

The difference between these is that in the first the gas is heated by a heat flux, while in the second by a light flux; furthermore, in the latter the ordinary speed of sound is to be taken into account instead of the isothermic.

The self-similar profiles of one of the intermediate solutions are shown in Fig. 2 for $Q_0=3.01$, $P_0=0.014$, and $V_1=1$. The recalculation for $Q_0=1$ yields: $P_0=0.0062$, $P_{max}=0.62$, $(PV)_{max} = 0.45$ and $V_1=1.31$.

The piston pressure $P(0)=P_0$ is the second parameter. Along with V_1 it defines the pattern of integral curves of Eqs. (1.8). Its increase results in a shift of the strong discontinuity, i.e., an increase of its self-similar coordinate x_2 and that of pressure P and velocity U throughout the interval $(0, x_1)$.

In conventional (adiabatic) gasdynamics the motion behind a shock wave does not affect the motion in front of it. However, in the presence of a light flux, perturbations are transmitted across the shock

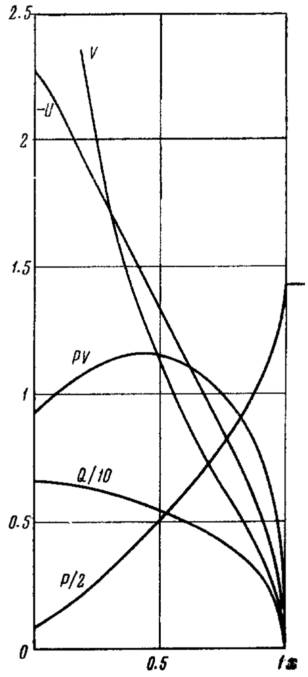

Fig. 1

wave. It should be noted that, as shown by numerical calculations, there exists for any mode at fixed V_1 a pressure P_*, such that the variation of the piston pressure P_0 in the interval $(0, P_*)$ has virtually no effect on the motion behind the shock wave. It is this aspect which makes it extremely difficult, when integrating Eqs. (1.8) from $x=x_1$ to $x=0$, to obtain the specified reasonably small pressure P_0.

In the presence of heat conduction, supplied heat flux F_0 is determined by the behavior of integral curves of Eqs. (1.7) as well as the parameters Q_0, P_0, and V_1. The case of $Q_0=0$ was considered in [6, 7]. The combined effect of fluxes Q_0 and F_0 does not materially affect the quantitative motion pattern considered here and in [6, 7]. The presence of heat conduction results in the smoothing of temperatures, particularly strongly evidenced by shock-wave isothermicity (see (4.1)). The combined effect of light and heat fluxes is shown for one of the modes in Fig. 3 for $\sigma=1$, $Q_0=12$, $V(x_1)=1$, $c=2$, and $d=-1$.

The dependence of the dimensionless maximum pressure P_{max} and temperature $(PV)_{max}$ on the dimensionless specific volume V_1 is shown in a logarithmic scale in Fig. 4 for dispersion into a vacuum ($P_0=0$) of a nonheat-conducting gas ($k_f=0$) with plasma coefficient k_q and $Q_0=1$. From these curves it is possible to determine at any moment of time t the maximum pressure $p_{max}(t)$ and the "temperature" $(pv)_{max}(t)$ for a given light flux $q(0, t)=q_0t^{3/2}$ and initial specific volume v_0

$$p_{max}(t) = tq_0^{3/4}K_q^{-1/8}P_{max},$$
$$(pv)_{max}(t) = tq_0^{1/2}K_q^{1/4}(PV)_{max}, \tag{7.1}$$

where P_{max} and $(PV)_{max}$ are functions of parameter

$$V_1 = v_0q_0^{1/4}K_q^{-3/8}.$$

From the results shown in Fig. 1, recalculated for $Q_0=1$, we obtain $P_{max}\approx 0.69$ and $(PV)_{max}\approx 0.44$ for $V_1 \ll 1$, while for $V_1 \gg 1$ from (6.6) we have

$$P_{max} = (2/3)^{2/5}V_1^{-7/5}, \qquad (PV)_{max} = (2/3)^{2/5}V_1^{-2/5}.$$

8. Let us consider a light flux $q(0, t)=q_0$ constant with respect to time, striking a matter of infinite density $(v(m, 0)=0)$. For the discharge of a nonheat-conducting gas into vacuum the initial and the boundary conditions for (1.1) are

$$u = v = p = 0 \quad \text{for } t = 0,$$
$$q = q_0, \quad p = 0 \quad \text{for } m = 0, \ t > 0. \tag{8.1}$$

The introduction of self-similar variables U, V, P, Q, and x by

$$u = t^{-1/c}q_0^{(a-b-1)/c}K_q^{-1/c}U(x),$$
$$v = t^{-3/c}q_0^{-(2b+1)/c}K_q^{-3/c}V(x),$$
$$p = t^{1/c}q_0^{(2a-1)/c}K_q^{1/c}P(x),$$
$$q = q_0Q(x), \quad x = mt^{(b-3a)/c}q_0^{-(a+b)/c}K_q^{-2/c}, \tag{8.2}$$

where $c=3a-b-2$, reduces (1.1) and (8.1) to the following system

$$rxU' = Uc^{-1} - P', \quad rxV' = 3V/c + U'$$
$$rx(VP' + \gamma PV') = (3\gamma - 1)c^{-1}PV - (\gamma - 1)Q'$$
$$Q' = -QV^aP^b, \quad r = (b - 3a)c^{-1}, \tag{8.3}$$

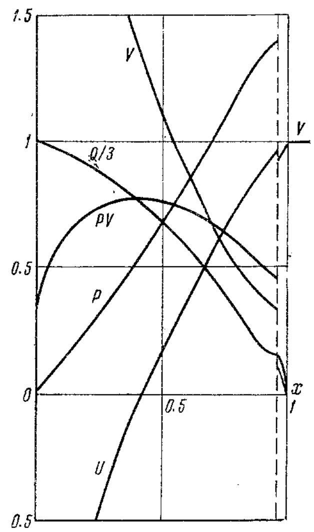

Fig. 2

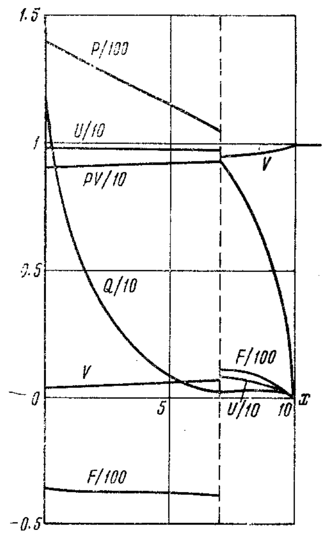

Fig. 3

with boundary conditions

$$P(0) = 0, \quad Q(0) = 1, \quad V = Q = U = 0 \quad \text{for} \quad x = x_1. \tag{8.4}$$

The two boundary points x=0 and $x=x_1$ are singular points of (8.3).

When inequalities $a < b < 0$ are satisfied in the vicinity of a point x=0 of the node kind, the solution can be expressed, to within terms of higher order of smallness, in the form

$$V = \left[\frac{(\gamma-1)(1-a)(xU_0/c)^{b-1}Q_0}{r\gamma(b-1) - [(3\gamma-1)/c - r](1-a)} + Ax^{\theta}\right]^{1/(1-a)},$$
$$P = \frac{U_0 x}{c}, \quad U = U_0 + \left[\frac{r(b-1) - 3(1-a)/c}{b-a}\right] xV,$$
$$Q = Q_0\left[1 - \left(\frac{U_0}{c}\right)^b x^{b+1} V^a \left(\frac{1-a}{b+1-2a}\right)\right], \quad \theta = \frac{(1-a)(3\gamma-1+3a-b)}{\gamma(b-3a)}, \tag{8.5}$$

where A is an arbitrary constant.

In the neighborhood of the singular point $x=x_1$, at which U=V=Q=0 and $P=P_1$, when the inequalities $r < 0$ and $a < 0$ are satisfied, the solution can be presented, to within terms of higher order of smallness, in the form

$$V = [aP_1^{6}(x - x_1)]^{-1/a}, \quad P = P_1 - r^2 x^2_1 V,$$
$$U = r x_1 V, \quad Q = \gamma r x_1 P_1 V / (1-\gamma). \tag{8.6}$$

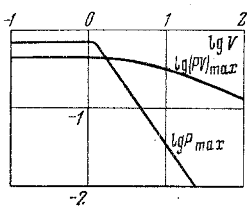

Fig. 4

Boundary-value problem (8.3)-(8.4) may be solved by integrating Eqs. (8.3) from x=0 to $x=x_1$ with the use of expansion (8.5), and selecting the free parameters A and U_0 so as to satisfy conditions $U_0=Q=0$ for V=0. The other method – integration of Eqs. (8.3) from $x=x_1$ to x=0, proceeding with (8.6) from the singular point x_1, and with parameter P_1 selected so as to satisfy condition P(0)=0 –was used in the derivation of results presented here.

The self-similar profiles shown in Figs. 5 and 6 were calculated for $\alpha=-5/2$, $b=-3/2$, and $\gamma=5/3$.

The coordinate of the singular point x_1 (the self-similar coordinate of the "vaporized" mass), the pressure P_1 (at the "solid body surface")

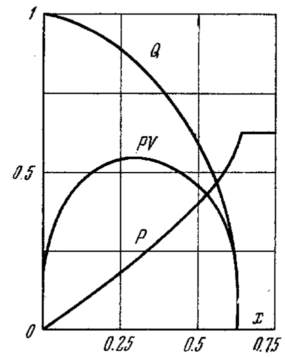

Fig. 5

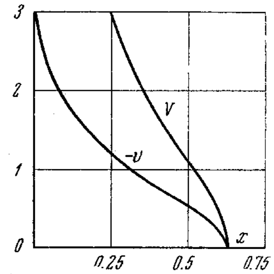

Fig. 6

TABLE 1

γ	5/3	7/5	6/5
x_1	0.630	0.543	0.425
P_1	0.637	0.517	0.367
$U(0)$	−3.38	−2.94	−1.40
x_m	0.286	0.249	0.107
$(PV)_m$	0.542	0.488	0.423
P_m	0.209	0.175	0.098
V_m	2.59	2.79	4.29
Q_m	0.849	0.859	0.972
U_m	−1.10	−1.02	−1.13

TABLE 2

γ	5/3	7/5	6/5
x_1	0.907	0.817	0.688
P_1	0.695	0.574	0.416
$U(0)$	−2.83	−2.41	−1.85
x_m	0.371	0.320	0.246
$(PV)_m$	0.423	0.364	0.277
P_m	0.240	0.193	0.130
V_m	1.77	1.89	2.12
Q_m	0.831	0.853	0.883
U_m	−0.961	−0.909	−0.832

at that point, the coordinate x_m at which "temperature" PV attains its maximum, and the values of all variables at that point, as well as the dispersion velocity of particles bordering on vacuum U(0) are given in Table 1 for $a=-5/2$ $b=-3/2$, and $\gamma=5/3$, 7/5, and 6/5, respectively.

In Table 2 the same magnitudes are given for $\alpha=-3/2$ and $b=-1/2$ (values typical for regions of multiple ionization [2]).

The dependence of maximum pressure p_m and maximum temperature $(pv)_m$ on time, on the numerical coefficient K_q, and on the intensity of the incident flux q_0, in the heating of an infinitely dense medium ($v_0=0$) with plasma coefficient k_p and $\gamma=5/3$ is defined by

$$p_m(t)=0.637t^{-1/8}q_0^{3/4}K_q^{-1/8},$$
$$(pv)_m(t)=0.542t^{1/4}q_0^{1/2}K_q^{1/4}. \qquad (8.7)$$

The author wishes to express his thanks to I. V. Nemchinov for suggesting the subject and for his valuable advice and discussion.

LITERATURE CITED

1. I. V. Nemchinov, "Certain non-stationary problems of radiative heat exchange," PMTF, vol. 1, no. 1, (1960).
2. I. V. Nemchinov, "Dispersion of a heated mass in an orderly mode," PMTF, vol. 5, no. 5 (1964).
3. Yu. V. Afanas'ev, V. M. Krol', O. N. Krokhin, and I. V. Nemchinov, "Gasdynamic processes in the heating of matter by laser radiation," PMM, vol. 30, no. 6 (1966).
4. N. G. Basov and O. N. Krokhin, "Conditions for heating of plasma by radiation from an optical generator," ZhÉTF, vol. 46, no. 1 (1964).
5. J. M. Dawson, "On the production of plasma by giant pulse lasers," Phys. Fluids, vol. 7, no. 7 (1964).

6. P. P. Volosevich, S. P. Kurdyumov, L. N. Busurina, and V. P. Krus, "Solution of one-dimensional piston motion in a perfect heat-conducting gas," Zh. Vychisl. Matem. i Matem. Fiz., vol. 3, no. 1 (1963).
7. V. E. Neuvazhaev, "Discharge of gas into a vacuum with the temperature at the boundary subject to an exponential law," PMM, vol. 30, no. 6 (1966).
8. Ya. B. Zel'dovich and A. S. Kompaneets, "On the theory of heat propagation under conditions of temperature-dependent thermal conductivity," in: Collection dedicated to the Academician A. F. Ioffe on his 70-th birthday [in Russian], Izd-vo AN SSSR, Moscow (1950), pp. 61-71.
9. Ya. B. Zel'dovich and Yu. P. Raizer, Physics of Shock Waves and High-temperature Hydrodynamic Phenomena [in Russian], Nauka, Moscow, Chapter 10, (1966), pp. 506-28.